GEOMETRY

BOOK I

Teachers' Guide for Class V

Experimental Edition

Prepared by
NCERT Mathematics Study Group

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PREFACE

For the last ten or twelve years a revolution, popularly known as the 'New Mathematics Revolution' has swept the world. It has involved an expenditure of millions of dollars, roubles, pounds, francs and rupees. What is more important, it has involved millions of man-hours of school teachers, college teachers, educational administrators and some of the foremost mathematicians of modern times. It has changed the attitude to mathematics of billions of children, teachers and parents.

About a hundred projects[*] have been set up all over the world for developing new curricular materials including text books, teachers guides, charts, models, film strips, films, T.V. lessons etc. and for training teachers in new ideas. In India, the efforts for improving school mathematics have included setting up a panel of distinguished mathematicians under the chairmanship of Prof. Ram Behari, an editorial board under my chief editorship, a project under the auspices of the Department of Science Education of the N.C.E.R.T., organisation of six Study Groups at Bangalore, Baroda, Delhi, Jaipur, Jadavpur and Kanpur, a large number of summer institutes for training of teachers and a national conference on school mathematics.[**]

The six study groups for developing new curricular materials in mathematics were set up in 1966 as a result of the decisions of the conference on mathematics and science education held under the chairmanship of Prof. D. S. Kothari. The coordinating committee of the Mathematics Study Groups decided to assign responsibilities as follows. Primary Mathematics (Class I—IV) and Applications of Mathematics to Kanpur Study Group, Algebra (including Arithmetic) for classes V to VII and VIII to X to Baroda, Delhi and Jaipur groups and Geometry (including coordinate geometry and trigonometry) for classes V to VII and VIII to X to Bangalore and Jadavpur groups. The entire work was to be coordinated by the coordinating committee consisting of the six Directors of the Study Groups.

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[*] Lockard D Report of the International Clearing House on Curricular Development in Science Education, Maryland University, U.S.A.

[**] Kapur, J. N Proceedings of the National Conference on School Mathematics, Mathematical Association of India.

So far the Study Groups have produced the following books:

(i) Handbooks for Teachers of classes I and II
(ii) Algebra text books and their teachers' guides for classes V, VI and VII
(iii) Geometry text books and their teachers' guides for classes V, VI and VII.

Experimental editions of these books are now being made available for try-outs in some selected schools. The final editions of these books improved in the light of the reactions of students and teachers will be made available for open use by state governments sometime in 1970. The present editions are for restricted use by authorised schools only.

Though Baroda, Delhi and Jaipur Groups worked jointly in writing the earlier versions of algebra books, the final version was prepared by the Jaipur Group. Throughout the preparation of the final version, the Jaipur Group was assisted by Shri P. K. Srinivasan of the Kanpur Study Group.

The final version of the geometry text-books has been prepared by the Bangalore Group and that of the geometry teachers' guides has been prepared by the Jadavpur Group.

In algebra, the approach has been through sets, truth sets of open sentences including inequalities and structures of the systems of natural numbers, integers and rational numbers. In geometry, the approach has been through transformations and symmetries. Both the approaches are likely to be new for teachers and therefore all efforts will be made to train the teachers in the new ideas. Even when training programmes are not available, it is hoped that the teachers' handbooks will be sufficiently lucid and clear to provide sufficient help to interested teachers.

Most of the ideas contained in these books have been tried out with many batches of students and the reaction of student and teachers has been of unmitigated enthusiasm. These were also discussed at the National workshop on school mathematics held at I.I.T., Kanpur in December 1968 and attended by representatives of eleven state institutes of education. Their enthusiastic support has been a source of strength to us.

I must take this opportunity of thanking Prof. D. S. Kothari, Chairman of the coordinating committee of all Science and Mathematics Study Groups, Shri L. S. Chandrakant, former Joint Director NCERT, Dr. S. K. Mitra, present Joint Director NCERT, Dr. M. C. Pant, Head of the Department of Science Education NCERT, Shri Rajendra Prasad, Field Officer NCERT, Shri D. Raghavan, Chief Publication Unit NCERT, Shri Chakravarty, Chief Production Officer NCERT, Shri N. A. Sriram of Bangalore Government Press for administrative support at all levels without which production and printing of these books would not have been possible.

On the academic side, I would like to thank all the Directors and members of the Study Groups whose names are given elsewhere for their cooperation. I must make special mention of Prof K. Venkatachalienger whose group prepared the first detailed curriculum of geometry and the first version of the geometry books, of Prof. G. C. Patni for giving valuable leadership to the Jaipur Group during the preparation of the final version of algebra books and of Dr. D. K. Sinha for getting geometry teachers' guides prepared.

I must thank all the managers of the presses for their sincere cooperation.

All the teachers are requested to fill in the proformas given in the teachers' guides and return these to the undersigned.

J. N. KAPUR
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I.I.T., Kanpur.

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SOME REMARKS ON THE TEACHERS' GUIDE

The mathematical ideas woven into the texture of the development set out in the textbook undoubtedly mark a departure from the traditional curriculum in geometry, to the one based on the study, from the standpoint of number (algebra) or as sets of points, lines etc and also from the standpoint of what have come to be known as transformations such as rotations and reflections. Transformation geometry (or motion geometry), as the following pages will bear out, apart from providing an excellent introduction to the study of geometry, unfolds the beauty and the fascination of geometry itself. Such an approach, it may, perhaps, be argued, goes a long way in putting a new slant to the teaching of geometry hitherto dominated by the traditional Euclidean style which, as we know, puts emphasis on rigid figures and as such, on the congruence of triangles. This study brings to the fore a number of geometric properties, but what is most important is that this study leads to some interesting elementary aspects of group theory and it is the idea of group of transformations or in a single word, *symmetry*, that permeates through the study carried out in the pages to follow. It is, in connection with the theory of groups, the study of geometry becomes immensely useful in crystallography and in many branches of science What the modern mathematics aims at is to unify algebra and geometry as one mathematics and not to treat them as its two mutually exclusive parts.

In this book, some broad suggestions are offered and they are intended to illuminate difficult parts of the new curriculum and also, to provide the teachers with what is happening beyond the curriculum followed in the text book. This guide has been arranged in chapters corresponding to those in the text book. In dealing with every chapter, we have not followed strictly some rigid set of rules, but, by and large, in each chapter, we have talked about

(i) the aims and objectives of the chapter,
(ii) the method of introducing concepts occurring in different sections of the chapter,

(iii) understanding and skills to be achieved,
(iv) class-room and student-activities including oral drills, if any,
(v) additional and special exercises, if any,
(vi) answers to the exercises in the textbook,
(vii) the whole list of concepts, subconcepts, vocabulary and symbols used in the chapter.

The discussions under the above heads are, inevitably sometimes more detailed and more discursive than others. These divisions, even though they exist in their own right, may not always be sharply drawn, and in fact, in many instances they do overlap. It is, therefore, advisable to look upon these separate discussions in a chapter as a *single* work, for it is then only, we can provide a coherent and developing account of what is styled as modern mathematics. The same arguments apply to each of the three parts of the books. The three books, though planned for three different classes, are closely related to each other and the topics have immediate relations to each other. It is, therefore, an advantage and as a matter of fact, the teacher can get the most out of it, if the teacher goes through all the books together with guides before embarking on a course with the Vth class. Besides, what have been stated above, the guide contains one article, on general remarks on the teaching of new mathematics. A list of books, journals, articles is given at the end of the book for study and for reference purposes, too. It is hoped the teachers' guide will contribute in some way to the unfolding of the essence of what is called 'new mathematics' and appreciating the train of thoughts in the minds of the authors of text books.

SOME GENERAL REMARKS ON THE TEACHING OF NEW MATHEMATICS

The only way, it seems, to make the teaching new materials to students effective is that a teacher of mathematics should participate actively in the proceedings of his class so that the students can discover the mathematical truth by themselves. In other words, the environment of the class should be so conditioned by the teacher that the students can draw upon the surrounding familiar objects or things so as to get at the truth intuitively and informally. It is obvious that no two teachers will explain and illustrate any new topic exactly in the same way, for the experience of one teachers' set of students is very likely to differ from that of the other teachers' set of students. It, therefore, follows that a topic need not be explained in toto as done in this guide, but certainly in keeping with the situation of the students and that may even vary from school to school. Thus the suggestions, contained in this book, serve as illustrations that the teacher may adapt to suit the needs of his particular class. The class-room situation, local happenings, the experience acquired in other science classes or elsewhere all together go to make up the totality of what is broadly called the experience of the students as referred to above and a teacher, if he is to present the new materials as something challenging and exciting, too, has to make use of this experience It is undoubtedly an onerous task, but once a teacher puts a slant to his teaching in terms of 'experience', 'activity' and 'discovery', he cannot merely stimulate curiosity of the young students but also can have the opportunity for tasting, in part, the joy of discovery, and pleasure of excitement of his students.

While introducing a new geometrical figure with a new vocabulary, the teacher should remember that recognition of the object should be acquired first by showing the objects or doing the action or using charts. It is, after that, the new vocabulary is to be introduced and the teacher should constantly use the word under different situations, so that the students can talk among themselves freely with the acquired vocabulary. It should be kept in our mind that the aim is to form associations, to link the topics with the things around the students until an use of them becomes effortless. This should form the perspective of the lessons to be framed by the teacher. It is for the teacher to decide how often the new concepts, questions and exercises should be repeated and what drilling should be undertaken. The teacher may find from time to time that some of the previous topics have not been fully understood and so, he should immediately revise and practise them, for unless this re-teaching is done now, the students will continue to harbour a wrong notion which may stand in the way of the introduction of a new topic at a later stage. Nevertheless, the teacher should never miss such an opportunity, for otherwise, it is then he can afford to recapitulate old things before he launches out to bring in new ideas. It may be suggested that a teacher while taking up a particular lesson, he may, by the way, introduce some of the concepts of the next lesson so that the teaching of the next lesson becomes more effective than what it would be, if some periods are exclusively allotted to it, as done at present. It is a fundamental principle of learning that the repetition of a topic with a new vocabulary in different situations and contexts is the key to its acquisition by the students and therefore, the teacher should make it a point to repeat the topic as many times as he can. This repetition and practice can be achieved by the use of questioning. Questions and the drills should be intimately associated with the experience of the pupils and should be illustrated in situations that have meaning for the pupils. It is only the teacher of the class who is in a position to know of the particular situation of his own class and perhaps, none else, not even the framer of the syllabus, the author of the guide-books or the textbook. Unless the teacher builds around the experience of the pupils, the drills that are not divorced from the situations and activities of life, he cannot expect his students to comprehend fully the materials he wants to drive home. In the course of teaching and writing in mathematics, a teacher can hardly afford to do without the use of blackboards. The class should be equipped with all the appliances, namely, a ruled blackboard, geometrical instruments, scissors, thin tracing papers, charts, models etc. that are essential for intensive class-room and student-activities.

In the choice of exercises, the teacher will have to exercise his own discretion and certainly he will choose those which he feels to be of greatest use to his students and of relevance to the topics concerned. Sometimes, he may have to provide simpler examples of some exercises or more examples to give further practice. It has to be remembered that although the textbook contains a number of exercises, the drilling may be oriented by the teacher in such a way as to elicit spontaneously some questions from the students themselves. In fact, the exercises that occur at the end of every topic (and sometimes at the end) should be thoroughly practised orally or otherwise, until the pupils can do them without mistakes. It may be pointed out that exercises should not be looked upon as tests. It is believed that intensive oral drill for sometime followed by written practice for another length of time every day will be more useful than using the whole of each period for oral drill and setting apart one whole period for writing. For a large class, when a teacher finds that most of the students have understood a point, after the thorough explanation and illustrations using the black-boards and questioning the students, he may divide the students into groups and may assign to the groups oral or written exercises. The written exercises may be done either on paper or on the blackboard. Some of the unfinished exercises may be assigned for home-work. Here, the teacher must bear it in mind that the pupils should never be overloaded with home-work of the drilling type, particularly at the elementary stage. It must be borne in our mind that home-work, more or less, should be *revisionary* in nature, so that the student can find time at home to master what he has learnt in the class. Thus, the students should have sufficient practice at home for the lessons done in the class. The teacher should see that the students do learn the notions properly and not have incorrect understanding.

CHAPTER I

INTRODUCTION

I. Aims and objectives of the Chapter:

The essential objective of this chapter is to introduce geometry on an intuitive and informal basis. As is well known, geometry is the mathematical model of the physical world around us and hence the chapter aims at acquainting the students with simple geometrical facts about the objects situated in the space. Thus, the student is gradually led, in an intuitive way, to some elementary concepts of geometry by means of examples that are drawn largely from his own surroundings This chapter also includes some basic facts about some undefined terms like point, line, plane, etc. The second objective of this chapter is to develop in the students, with the help of appropriate illustrations, an appreciation of the relationships between the different figures of space and learn the basic incidence properties. It is a part of the aim of this chapter to introduce, again through examples, the idea of parallel lines and planes.

II Method of introducing concepts:

General notes: There are a number of new concepts and subconcepts to be introduced in this chapter. The method of introduction of such vital concepts to tender minds needs careful and judicious steps on the part of the teacher. The crux of the method lies in presenting the concepts in terms of what have come to be known as 'activity' and 'experience', rather than of knowledge to be acquired and information to be stored and memorized. It is to be remembered that copious examples stimulating intuition will help the teacher to make the children learn a fact and, in fact, a teacher has to use his imagination in selecting appropriate examples so as to bring out the relationships between the geometrical objects. As a matter of fact, we may regard geometry treated on an intuitive basis as the preliminary analysis of the physical geometry of the space in which we live. This approach will lead subsequently to the idea of a conceptual world to correspond to the physical world and thus, the initiation to the structure of the abstract mathematical system may become smoother This intuitive way of introducing geometry will lead, at a later stage, to a logical structure of geometry as a deductive system. One has naturally to start with certain undefined terms, which can be intuitively grasped by the children. We are interested in the mutual relations between these and the teacher must make use of every opportunity to point out the mutual relations which form a part of the experience and activity of the children

SECTION 1.2: *Recognition of shapes and sizes of geometrical figures*:

To begin with, a set of models (both wire and solid) of geometrical objects such as the cube, regular tetrahedron, rectangular parallelopiped, sphere, hemisphere, cone, cylinder, prism, pyramid, other regular polyhedra, and of geometrical plane figures such as triangle, parallelogram, rhombus, rectangle, square, pentagon, disc etc., may be placed in full view of the students. The set should contain a number of similarly looking figures, some of them being of the same size and some of different sizes. The difference in size should be such that a student of this age may distinguish them by observation. The students may be divided into groups of four or five and each group may be provided with a set of plastic or card board models of figures, as those shown on a chart (Fig 2 & 2a). They may be asked to arrange them as indicated in the table in page 4 of the text book. They may also be provided with charts as in (Fig. 2 & 2a) and may be asked to shade the figures by coloured pencils, using one colour for similarly looking figures. It is to be borne in mind that at this stage it will suffice if the students can distinguish the shapes and sizes of different figures. It is not desirable to ask the students to memorize the names of the figures or objects.

As regards Example 2 Exercise 1.1, cutting may be done by the teacher. The students may, then, be asked to reassemble them to make the figures (fig. 3). This exercise will be an interesting activity on the part of the students.

SECTION 1 3: *Point, line, plane*:

We have already mentioned that one has to start with some undefined terms. These terms are point, line and plane. It has also been pointed out that these terms can be intuitively grasped by the children. Suitable illustrations from the objects around us should be chosen to give an idea of these undefined terms.

Point: The point may simply be explained by putting a dot on the blackboard. The dots on the blackboard (or on a paper) have to cover some area, otherwise they become invisible. It is evident that there is an underlying idea of approximation and idealisation in the grasping of the notion of a point from the pictorised dots. This may be explained easily by mentioning that the stars [e g., Betelgeuse (Ardai), Antanes (Jycohota)] are nearly thousands of times larger than the sun, but in comparison to the sun, which appears as a disc, the stars look like points, as the sun is comparatively nearer to the earth.

The teacher may, however, draw dots of smaller and smaller sizes on the blackboard to get the idea of a point. The students may be asked to do the same on a paper by a sharp pencil. They may be asked to give examples of points found in surroundings familiar to them (Exercise 1.4, No. 1.)

Line: The fact that the walls in the class room or the faces of a box meet in a line may be used to explain what is called a line. Lines may be illustrated by paperfolding (using thinner and thinner sheets of paper) . The use of straight edge or ruler to draw lines (sec. 1.10) may be taken up here. The students should have sufficient practice of drawing lines by using straight edge or ruler. The students may be asked to take two points on a paper and to fold the paper through these points. They may also be asked to draw a line through these points by a straight edge. They will find that the paper may be folded in one way only or only one line can be drawn through two points. The idea that a line extends indefinitely on both ways may be explained clearly to the students by taking larger and larger sheets of paper. The distinction between lines (straight lines) and curves (curved lines) may be illustrated by taking a piece of 50 paise or similar things.

Plane: The surface of a glass pane or the blackboard may be considered as a plane. The floor, walls, roof of the classroom, the top of the teacher's desk, the sheet of paper, the playground may furnish the examples of a plane. The idea that a plane extends indefinitely on all sides may be made clear to the students. The idea of a curved surface may be illustrated by a ball, a glass, a spoon etc.

By placing a pencil on a paper the characteristic property of the plane that the line joining two points on it lies wholly on the plane, may be explained to the students.

Lines and planes may be looked upon from another point of view By drawing a line we can mark any number of points on it, so that a line may be considered as a collection of points or using the set language, a line is a set of points (If A is a point on a line l, we say that $A \in l$ or $l \ni A$). On a plane also we can get a line by joining any two points ; we can consider a plane to be a set of lines as well as a set of points Each of the lines on the plane may be considered as a set of points, which also belong to the plane. Therefore a line may be considered as a subset of the points of the plane. (If a point A is on a line l and the line l is on a plane p, then $A \in l \Rightarrow A \in p$).

Exercise 1.4, No. 1 may be taken up here.

Concepts: Point, line, plane.

SECTION 1 5: *Recognition of some simple plane figures and regular solids.*

Our object is to put some sense and order into the objects which we all see around us. It is not intended, at this stage, to make the students know all the properties of these objects.

We shall consider the plane figures such as triangle, quadrangle, pentagon, hexagon and some simple solids such as cube, cuboid, regular tetrahedron, etc. and see how many sides, vertices, faces and edges they have.

For this purpose we have to introduce the idea of segment. The terms 'open segment', 'closed segment' may be taken up afterwards. However a point may arise in the mind of the students whether the points A and B are included or not. At this stage, by the term segment a closed segment may be understood and therefore, the points A and B are included.

In a ray, that the initial point is included must be specially mentioned.

The students then should be provided with the models of the plane figures mentioned above and may be asked to count the number of sides and vertices each figure has.

We then pick up some simple solids as mentioned above and perspective drawings of each of the figures should be drawn on the boards. It is better to use charts and models. In this case also, students should be supplied with the models and they should be given the opportunity to 'discover' the number of bounded faces, number of edges, number of corners each solid has. (Of course, what we mean by the terms face, edge, corner have to be explained to the students first). Illustrations of the solids may be picked up from the objects familiar to the students. For example, cube may be illustrated by sugar cube, cubical picture, blocks, cuboid by a rectangular box (brick is not a good example, for, at least one of the faces is not completely plane), sphere by cricket ball, foot ball, rubber ball etc. cylinders and prisms by pencils and cone by heaps of sand, sugar etc.

The students may be provided with models of square, rectangle, triangle, hexagon, disc etc. and may be asked to find out the resemblance of these figures with the faces of the solids. Thus, they will find for themselves that each plane face of a cube is a square, that of a cuboid a rectangle, the faces of a tetrahedron are triangles, two faces of a cylinder are discs etc. (The idea of square and rectangle may also be made clear by the page of a book, windows, floor, walls etc.).

It will be interesting for the students to recapitulate the number of faces, edges, corners of the solid figures and to find which obey the rule:

No. of faces + No. of corners = No. of edges + 2 [Descartes — Euler—Poincaré]

In Exercise 1.3 the symbols N_0, N_1, N_2 have been used to give an idea of the dimension of corner, edge and face respectively, by the suffixes 0, 1, 2. Any other set of symbols may also be used.

Nos. 2-6 of Exercise 1.4 may be taken up here. As we have already mentioned that the students need not memorize the names of the geometric objects, in case of Ex. 1.4 No. 2 the models of some geometrical objects may be placed on the table and the students may be asked to point out the objects (models) which are of the same shape with the objects in question. In case of other exercises also the models may be supplied

SECTIONS 1.4 and 1.6: *Collinearity, concurrency, plane and space configurations.*

The students may be asked to take a number of points at random on a plane and then to draw lines through those points. They will find that in some cases a number of points may be in one line. Those points are collinear. The students may be asked to draw a line on the paper and to mark a number of points on that line. They will find that these points are collinear, i.e., the set of all points on a line are collinear. A network of fine wire may be shown to the students to find the collinear points.

The students may be asked to take a point on a paper and to fold the paper in different ways such that all the creases contain that point. These are examples of concurrent lines. Concurrence may be illustrated by a box or a room where three edges concur at each corner point.

The complete quadrilateral and Desargues configuration (both coplanar and non-coplanar) may be demonstrated to the students and the students may be taught to prepare the model by tying 10 cycle spokes with a string or thin wire.

Subconcepts: Collinearity of points and concurrency of lines.

SECTION 1.7: *Parallel lines*:

The next topic is concerned with parallelism of lines in a plane. To begin with the discusion of parallel lines, the students may be told what we mean by parallel lines and in fact, such statements have to be explained by drawing figures, and taking examples from the physical space around us, so that the students might have a feeling about these things which are intuitively very simple. It may be explained to the students that a pair of lines lying in a plane drawn in the same direction (taken in an intuitive sense) do not meet at all and we call such lines to be parallel to each other. It may be pointed out that the things that we see often, i.e., the opposite sides of door frames, opposite edges of carpets, the railway lines (straight structures) are examples of pairs of parallel lines.

The parallel ruler and set-squares may be used, at this stage, for drawing parallel lines (sections 1.11 and 1.12). The parallel ruler may be employed for drawing a pair of parallel lines. It may also be employed for drawing a line parallel to a line through a given point. But when the point is very close to the line or very far off, the parallel ruler cannot be used. In such cases a ruler and a set-square may be conveniently used. This can easily be done by putting the edge of the ruler along the given line and the set-square by its side, so that one of its edges, which are at right angles, along the ruler. The set-square is now moved so that the other edge passes through the given point. A line is drawn along the edge. This line is parallel to the given line. The students may be asked to draw parallel lines. The students may be asked, at this stage, to see how many lines may be drawn through a point parallel to the given line. This will lead the students to get at the truth of parallel axiom, which states that 'through a point not on a line, there is one and only one line parallel to the given line.'

The students may then draw several bundles of parallel lines through different points on a certain line. It may be explained to the students that all lines in each of them point in the same direction. The teacher should also draw lines in the plane of the board in the directions of East, West, North, South, North-East etc. (corresponding to a wall map hung on the board) and the idea of parallel lines can thereby be made clear. In whatever direction, we draw a bundle of parallel lines, it is easy to see that the entire plane can be covered by a set of parallel lines. This is what is known as the partition of the plane. For different directions, we will get the different partitions of the plane.

An interesting construction may be demonstrated to the students. By drawing two sets of parallel lines in different directions, we construct what is called a net. By joining corresponding points of intersections of the net other sets of parallel lines may be drawn. The students may be asked to draw these sets of parallel lines. This may be realised in practice by the floor of a room covered with tiles which are rectangular in shapes or with a carpet. It may be left to the students to discover that each constituent figure has its opposite sides parallel and each of them is a parallelogram. At this stage the teacher may show the method of drawing a parallelogram (section 1.13) . A ruler with two parallel edges may be used in constructing parallelogram (also a rhombus) by placing the ruler in two different positions. The students may be asked to cite examples of parallel lines from the surroundings.

Concept: Parallelism

SECTION 1.8: *Horizontal plane, Horizontal line, Vertical line, Vertical plane.*

Horizontal plane may be illustrated by putting water or spirit in a vessel We note that the surface of water in whatever manner we may keep the vessel is an example of a horizontal plane. Every line in a horizontal plane is called a horizontal line. The vertical direction may be exemplified by using a plumb line, and by a thin rod placed with one of its ends just above horizontal liquid surface which when reflected in the horizontal liquid surface coincides with the direction of the rod produced in the liquid. A vertical plane is a plane which contains at least one vertical line. All of them may be illustrated by the faces of a rectangular box. The examples of floor, roof, walls of a room may be made use of to point out the properties that (a) all horizontal planes are parallel (floor and roof of the room); this property may also be illustrated by taking liquids in different vessels at different levels. (b) all vertical lines are parallel (edges where the walls meet); this may also be illustrated by two plumb lines at different points and again two plumb lines at same point. (c) horizontal lines may intersect (edges where two adjacent walls meet the floor), (d) the vertical planes may intersect (two adjacent walls may intersect in a line).

Ex. 7 of Exercise 1.4 may be taken up here.

Subconcepts: Horizontal and vertical lines, horizontal and vertical planes, parallel planes.

SECTION 1.9: *Incidence properties*:

We will now recapitulate the concepts of points, lines, planes and try to obtain some important relations between them. These relations are called *incidence properties*.

We have alredy seen that we can draw a line joining two points, We have also seen that two lines in a plane may intersect in a point or not When they do not intersect they are parallel. By drawing various pairs of lines in a plane we can easily see that no pair intersect each other in more than one point. We may, thus, say when two points are in common, the lines are not distinct, that is, we can draw exactly one line joining two points in a plane. Thus we get the following two incidence properties in a plane:

(1) There is exactly one line which joins two points in a plane.

(2) Two lines in a plane either have a point in common or are parallel.

The relation between a line and a plane may be illustrated by means of a pencil and the board or the surface of the table. The pencil may be kept on the table (1) wholly lying on the table (2) with one end in contact with the table or (3) in such a way that it does not meet the plane of the table even if the pencil is produced both ways. This will lead us to the following incidence properties in space:

(1) A line may have exactly one point of intersection with a plane.

(2) Two points determine a line and if two points of a line lie on a plane then all points on the line lie on the plane.

(3) A line may not have a common point with a plane Then the line is said to be parallel to the plane

We have also seen that the planes may or may not intersect, for example, the adjacent walls of a room or the roof and the walls or the floor and the walls meet in a line, but the opposite walls of a room or the floor and the roof do not meet.

Thus we have the other property,

(4) Two planes are either parallel or intersect in a line.

We have noticed that two lines in a plane either intersect or be parallel. Therefore, we can draw a plane through two intersecting or parallel lines. Again we have seen that two planes have no line in common when they are parallel or they may have one line in common when they intersect each other, but they cannot have two lines in common. Therefore, exactly one plane can be drawn through two intersecting or parallel lines.

Ex. 7(9) of Exercise 1.4 and Ex. 2 of Exercise 1.5 may be asked to the students at this stage.

III. Understanding and skills to be achieved :

It needs hardly be stressed that the main object in the teaching of any topic should be such that the children can have a clear grasp and understanding of what he has learnt and can have the ability to discover things for themselves to solve exercises ; and can find interest in the things taught to them It has to be remembered that understanding has to grow gradually from the experiences of the child, particularly at this elementary level.

One purpose of this chapter is to provide students with an understanding of important relationships of objects in space. The student has to comprehend fully the geometric information about different figures in space and he should be able to solve relevant exercises involving these relationships. The drilling has necessarily to be such that the students are able to acquire a sense of geometric forms, their mutual relations and also skills in simple geometrical constructions. To be more specific, the students should achieve the skill of drawing parallel lines, and should see for themselves where and how they occur, how they are related to each other in the simple figures both plane and solid around them.

IV. Class-room and Student activities :

There is no denying the fact that in keeping with the elan of new mathematics, the children have to participate actively in the teaching in the class-room along with the teacher so that they gradually develop a working facility in the objects to be learnt. Moreover, this makes the task of the teacher more pleasant. The only way to foster active learning and thus, to stimulate the curiosity of the student is that a teacher should explain an idea by means of as many different descriptions as he can and should also make the environment of the class lively so that he can share with his students some of their mathematical experiences.

As for the purpose of demonstration, in this chapter, the teacher requires (i) black-board (unruled), (ii) black-board ruled into squares, (iii) black-board instruments: straight edge, compasses, protractors, (iv) models and charts, (v) scissors, (vi) peg-board or pin board, (vii) set-squares, (vii) parallel ruler, (ix) link-parallelogram, (x) chalks coloured and white.

The apparatus necessary on the part of the students consists of (i) set-square, (ii) ruler, (ii) scissors, (iv) plain paper for folding, (v) graph paper, (vi) parallel ruler.

V. Answers to the exercises in the book :

Exercise 1.1

Ex 1.	Figure	A similar looking figure of the same size	A bigger or smaller similar looking figure
	1	12	8
	6	14	11
	2	13	10
	15	21	5
	7	9	19
	3	17	16
	4	18	20
	22	33	32
	23	37	31
	30	39	34
	25	28	36
	24	35	27
	26	38	29

Ex. 1.3	No. of vertices N_0	No. of edges N_1	No. of faces N_2	$N_0 + N_2$	$N_1 + 2$
1. Tetrahedron	4	6	4	8	8
2. Cuboid	8	12	6	14	14
3. Octahedron	6	12	8	14	14

Ex. (P 17)

3 : C, C′, O
4 : L, B, C
5 : M, A, C
6 : M, A′, C′
7 : L, B′, C′
8 : N, A, B
9 · N, A′, B′
10 : L, M, N

Exercise 1.4

(1) Plane — pages of a book, top of a table, black board, glass pane, floor.

Line — crease of a paper fold, the intersection of floor and wall, railway line (straight structure), edges of a box, fine needle.

Point — corners of box, tip of a needle, pin prick in a paper, stars, corners of a black board.

(2) (a) rectangular parallelopiped or cuboid.
(b) sphere (c) cylinder or prism (depends on the pencil shown) (d) disc.

(3) (a) 6 (b) 4 (c) 8.

(4) tennis ball, rubber ball, marble etc.

(5) 12, 8, 3.

(6) (1) None, (2) None, (3) Yes, (4) No.

(7) (1) Yes, because a pair of parallel lines on a horizontal plane are both horizontal.

(2) No, because all vertical lines are perpendicular to any horizontal plane.

(3) Yes, because all vertical lines are perpendicular to same horizontal plane.

(4) No, because any two intersecting lines in a horizontal plane are horizontal lines.

(5) Never.
(6) Never.
(7) Never.
(9) (a) point (b) line (10) always (11) No (12) No (13) Yes (14) No (15) Yes (16) Yes (17) No

Exercise 1.5

(1) (a) 8, (b) 6, (c) 12, (d) square, (e) 2.
(2) There are three relations that can exist:—
  (i) A line can intersect a plane in one point.
  (ii) A line can be parallel to a plane.
  (iii) A line can lie wholly in a plane.
(4) line of intersection of any wall with floor or roof and line of intersection of opposite wall and any adjacent wall.

VI. List of concepts, sub-concepts, vocabulary, symbols

Concepts: Point, line, plane, parallelism, incidence properties.

Subconcepts: Horizontal and vertical lines, horizontal and vertical planes, collinearity of points, concurrence of lines, parallel lines, parallel planes.

Vocabulary: Point, Line, Plane, Cube, Cuboid, Regular tetrahedron, Rectangular parallelopiped, Octahedron, Square, Rectangle, Parallelogram, Rhombus, Triangle, Pentagon, Hexagon, Prism, Pyramid, Cone, Cylinder, Sphere, Disc., Edges, Corners, Faces, Vertices, Parallel ruler, Curve, Collinearity, Concurrence, Segment, Ray, Sides, Complete quadrilateral, Desargue Configuration, Symmetry, Axiom, Partition, Incidence properties, Set Squares, Theorem.

Symbols:

(1) $\in$ (Belongs to)
(2) $\ni$ (Contains)
(3) $\parallel$ (Parallel)
(4) $\cap$ (Intersection)
(5) $\Rightarrow$ (Implies).

CHAPTER II

REFLECTION MAPPING

I. Aims and objectives of the chapter

In this chapter, more basic mapping viz., reflection in a line (or plane) is employed. Since any number of reflections performed successively in an even number of lines is either a *rotation* or *translation* we realise the same Euclidean Geometry. In this chapter, reflection in a line has been easily illustrated by the use of mirrors and paper folding; this enables children to understand perpendicularity in an easy and intuitive manner, and offers opportunities to stimulate children-activity. And this leads to the use of Euclidean instruments viz., ruler and compass in principles of geometric constructions.

II Method of introducing concepts

General notes: In the previous chapter the incidence relations between points, lines and planes have been explained. We next take up the basic transformations or mappings which are associated with the physical space in which we live (Euclidean Geometry), [Euclids, as is well known, "flourishing ruler and compass so deftly moves in the realm of geometric figures" to quote the words of the great Mathematician Hermann Weyl vide Classical Groups (Princeton) p. 273]. In other words Euclidean Geometry uses congruent transformations or mappings (rotation and translation) which preserve the size and shape of geometrical objects around us. It will be our next objective to lay down the foundation on which the superstructure of 'transformation geometry' is to be built. The students may be asked to find by themselves what happens when a figure undergoes several transformations. The reflection in a mirror is the basic elementary transformation on which the development of elementary geometry proceeds; corresponding to every object a mirror provides a unique image; this image when reflected in the mirror becomes the object itself. This, obviously, as the teacher is aware, is three-dimensional in nature and to make it easier for the students to comprehend, one has to restrict the work to two-dimensions. This may also be illustrated by paper folding.

SECTION 2.1: *Mapping*

The concept of 'Mapping' is too difficult to be conceived by the students at this stage. It is to be introduced in an informal way only by some illustrations, such as, representation of the world by a globe, copying of the map of India in the students' drawing book from the wall map, which is a mapping of the sheet on the wall on to a page in the drawing book.

We have seen in the previous chapter that a plane is a set of points. If we now transform the points of the plane by some kind of transformation so that the transformed points lie on the original plane, then the plane which contains the set of transformed points will coincide with the original plane. We will say that this is a mapping which maps a plane on to itself

SECTION 2.2: *Symmetry in a line*

The property mentioned above is observed in the case of reflection. The fact, that corresponding to every object a mirror provides a unique image and this image when reflected in the mirror becomes the object itself, should be demonstrated by the teacher using small mirror strips (preferably doubled ones) and long needles fixed to corks and by the method of paper folding. The students may be asked to draw a line and to mark a small ink dot on a sheet of paper and to fold the paper along the line. Another ink dot will be obtained thus The latter point is the image of the former one. They may be asked to repeat the method for a large number of points taken on both sides of the line. This will lead the students to conclude that every point on the plane of the paper will go to another point on the plane of the paper when reflected in the line. That is, the reflection mapping in a line l of the plane is a mapping of the plane on to itself. The reflection mapping can be written symbolically as $P' = \lambda(P)$ where P' is the image of P when reflected in a line l (in the text book, the points have been denoted by capital English letters, line by small English letters and reflection mapping by corresponding Greek letters. The teacher may, however, use any suitable symbols). We can write the previous result symbolically as $\pi=\lambda\ (\pi)$ where π denotes the plane.

Fixed points for a reflection in a line:

While performing the above experiment the students will observe another interesting property of the reflection mapping. Let some dots be marked on the line of reflection (line may be drawn by a pencil) and then the paper be folded along the line, it will be observed that there will be no new ink marks on the paper, corresponding to these ink dots i.e. the image of each of the points on the line will coincide with the points. These points are called fixed points ; symbolically we write $P \in l \Longleftrightarrow P = \lambda\ (P)$

The symmetry line of two points:

The method of paper folding reveals that if P′ is the image of P, then P will be the image of P′. This may be made clear to students in the following way. The students may be asked to mark a dot (P) by blue ink on the paper and to get the corresponding ink dot (P′) by paper folding. They may be asked then to give a dot by red ink at the latter position (P′) and to get the corresponding ink dot (red mark) . They will observe that the coresponding ink dot will be at (P). These two points are called symmetric with respect to the line This line is called the line of symmetry. The line of symmetry for two given points A and B may be obtained by folding the paper in such a way that A and B coincide. This can be done by pricking a pin through A and B and then folding the paper to get the crease. Now the students may be given the idea of symmetric figures. For these figures, there is one-to-one correspondence in the reflection mapping i.e. the figures will map on to itself when reflected in a line (or in a point, which will be discussed in class VII). We may say, therefore, that a symmetric figure is unaltered by reflection in a line. This may be illustrated by simple symmetric figures which are familiar to the students, viz., a flower vase, a kite, an aeroplane, a leaf etc. The students may then be asked to discover for themselves more such symmetric figures with the help of paper folding; semi-transparent papers may be used for this purpose. The figures may be drawn with deep lines so that, when folded, the lines may be seen. The students may then be provided with one half of a symmetric figure and to draw the other half. Simple figures may be taken for this purpose. This may also be done with the help of semi-transparent papers, carbon paper or a kind of soap specially meant for this purpose.

No. 1-6 of exercises 2.1 may be taken up here.

Concepts: Reflection mapping, symmetry in a line.

Sub-concept: Fixed points.

SECTIONS 2.3 AND 2.4. *Perpendicularity. Reflection of a line in a line: Fixed lines.*

Section 2.4, which deals with fixed lines, may be discussed before discussing the section 2.3, which deals with perpendicularity, because the idea of fixed lines will be required to give a clear conception of perpendicularity.

We have seen that on the line *l* each point will remain unaltered when reflected in *l*. This means that the line *l* maps on to itself by the reflection in the line *l*. We now fold the paper about the line *m* through a point not on the line *l* and its image. We take a large number of ink dots on this line on both sides of *l*. (To distinguish, the ink dots may be red on one side and blue on the other) . The paper is now folded along the line *l*. It will be observed that the images of all the points will be on the line *m*. We, therefore, conclude that the line *m* is also mapped onto itself by reflection in *l*. The line *m* is called a fixed line. It may be pointed out to the students that all the points on the line *l* are fixed, but the points on *m* are not fixed except the point in common with the line *l* (i.e. the point of intersection of *l* and *m*). It may also be pointed out to the students that if we take another point neither on *l* nor on *m*, we can get another line through this point and its image. This line also will be a fixed line. Thus we may get as many fixed lines as we please.

We have observed that *l* is a fixed line for reflection in *l* and *m* which is distinct from *l* is also a fixed line. The line *m* is said to be perpendicular to *l*. The students may be asked to fold the paper about the line m and to see what happens. They will be able to discover for themselves that m, l are also fixed lines for reflection in m, i.e. l is perpendicular to m. Symbolically, we can write $l \perp m \Leftrightarrow m \perp l$. The students may be asked to cite examples of perpendicular lines from physical situations.

Concepts: Fixed lines, perpendicularity.

SECTION 2.5: *Drawing perpendicular to a given line*:

The point may be on the line or outside the line. The paper can be so folded that the crease passes through the given point and the line falls on itself. The crease (line) is then perpendicular to the given line through the given point.

Set squares may be used to draw perpendiculars. We put the edge of a ruler or a set square along the line and then place another set square with one of its edges, which are at right angles, in contact with the ruler or the square put along the line. We, then, slide the latter set square keeping the other fixed so that the other edge passes through the given point. We draw a line along this edge. The line obtained is perpendicular to the given line through the given point. It is obvious that we cannot draw more than one line perpendicular to a given line through a given point. For, we can fold the paper through a point so that the line falls on itself in one and only one way. The students may be asked to obtain this property by themselves.

Perpendicularity and parallelism

Since we can draw one and only one line perpendicular to a given line through a given point, any two lines perpendicular to the given line will have no point in common, i.e. they are parallel. Symbolically, we may write

$$m \perp l \text{ and } n \perp l \Rightarrow m \parallel n$$

Actually this property has been used to draw by set squares a line parallel to another line through a given point. (This property provides us with another definition of parallel lines which may be stated as follows: Two lines are said to be parallel if each of them is perpendicular to a line).

The students may, be asked to practise the drawing of perpendicular lines.

Sub-concept: Relation between perpendicularity and parallelism.

SECTION 2.6 *Reflection of a line: Reflection of parallel lines and perpendicular lines*:

If we take two points on a line *m* and the points are reflected in another line *l*, then the line *m′* joining the images will be the reflection of the line *m*. This may be demonstrated to the students by taking a large number of ink dots on the line *m* and folding the paper about the line *l*. There will be corresponding ink dots on the line *m′*. Symbolically, we write if λ stands for the reflection in *l*, then

$$A \to \lambda(A), \quad B \to \lambda(B) \to \overleftrightarrow{AB} \Rightarrow \overleftrightarrow{[\lambda(A), \lambda(B)]}$$

that is, the line $\overleftrightarrow{[\lambda(A), \lambda(B)]}$ is the reflection of the line $\overleftrightarrow{AB}$. (Symbolical expressions may be made use of, but need not be insisted upon from the students).

Another important property of reflection mapping is that two perpendicular lines are mapped again into two perpendicular lines by any reflection. The students may be asked to observe by means of paper folding whether the lines obtained are perpendicular. In a similar way, we can find the property that parallel lines remain parallel even after one reflection (and therefore after any number of reflections performed in succession in any order).

SECTION 2.8 *Congruent segments*:

The student may be asked to take two points, A, B, on a paper and then to draw 4 or 5 lines on one side of the points. He may, then, be asked to fold the paper about the first line and to get the images A_1, B_1 of A_1, B respectively. The paper may again be folded about the second line to get the images A_2, B_2 of A_1, B_1 and so on. We say that the two segments A B and A_1 B_1 are congruent to each other, which we write as $\overline{AB} = \overline{A_1B_1}$

and hence $\overline{A_1B_1} = \overline{A_2B_2} = \ldots\ldots\ldots\ldots$

If a, b, c, d, … … …… . …. .are the lines and α, β, σ, δ, denote the reflection mapping in the respective lines. Then

$\overline{A_1B_1} = \alpha\ \overline{(AB)}$, $\overline{A_2B_2} = \beta\ \overline{(A_1B_1)} = \beta\ \alpha\ \overline{(AB)}$, $\overline{A_3B_3} = \sigma\beta\alpha\ \overline{(AB)}$, $\overline{A_4B_4} = \delta\ \sigma\ \beta\ \alpha\ \overline{(AB)}$ . . ..
i.e., $\overline{AB} = \overline{A_1B_1} = \overline{A_2B_2} = \overline{A_3B_3} = \overline{A_4B_4}$ ....
This leads to the definition of congruent segments.

By folding the paper in the reverse order it can be shown that

$$\overline{AB} = \alpha\ \beta\ \sigma\ \delta\ \overline{(A_4B_4)}$$

Concept: Congruence.

SECTION 2.9: *The pair of compasses and the Dividers.*

The use of these instruments to lay off segments congruent to a given segment on given lines may be demonstrated to the students The students also may be asked to practise the same.

The procedure we have adopted earlier for obtaining the reflection of any figure is the method of paper folding The use of the dividers and the pair of compasses to lay off congruent segments provides us with another procedure for constructing the reflections of any figure in a line. We know that the image of a point lies on the line perpendicular to the line of reflection and it can be shown that the image distance is equal to the object distance. Any figure is a set of points. Therefore, we may construct the reflection of a figure in a line in the following way. We draw lines through the points on the figure perpendicular to the line of reflection. We then lay off on each of this perpendicular line a segment (starting from the point of intersection) congruent to the segment from the point on the figure to the point of intersection on the side opposite to the figure. We will obtain the image of the figure by joining the end points of these segments.

SECTION 2.10 & 2.11: *The circle, intersection of a line and a circle.*

The last topic in this chapter is the circle. The students may be taught to use compasses to describe a circle . The students may be asked to draw a number of circles with different centres and radii of different lengths. They may also be asked to draw circles with the same centres and same radii. This will lead to the conclusion that we can draw one and only one circle of a given radius with the given point as centre. The interior and exterior of a circle, chord, diameter, segment of a circle, arc etc. may be illustrated with good figures. This property that a circle is mapped on to itself by a reflection in any diameter can be verified by paper folding.

The students may be asked to draw a circle on the paper and to take any point outside the circle. They may be asked to draw a number of lines through this point. It will be observed that some lines will not intersect the circle, some will intersect at two points and at most two lines may be obtained to meet the circle in only one point. These lines are tangents to the circle.

The students may be asked oral questions on the definitions of diameter etc. and to draw a number of circles.

Concept: Circle.

Subconcepts: Radius, diameter, tangent etc.

III. Understandings and skills to be achieved

The topics on reflection and symmetry are not usually found in the traditional curriculum in school geometry; the teacher must make every endeavour to direct the students' attention gradually in an intuitive manner to the relevant properties that remain invariant when the figures undergo transformations.

The student should get sufficient practice of drawing perpendicular lines, circles, and should be in a position to recognise the various elements connected with the circles. Finally, the students should acquire a command over vocabulary and the corresponding notations employed so that they may express clearly what they have learnt and this is vital to the understanding of any topic specially in mathematics and science.

IV Class-room and student-activities

The reflection mapping as an abstract concept can be understood in the correct sense initially by the concrete use of the mirror and the paper folding. In this connection, the teacher should draw on a paper the pictures of a number of symmetrical figures with which the students are familiar viz. a flower vase, a kite, an aeroplane, a leaf and then ask the students to discover for themselves more such figures with the help of paper folding (semi-transparent paper). After this, the teacher will take a rectilinear figure on one side of a line and will draw its image on the other side. To explain the line of symmetry to students, the teacher may take the diagram of a kite and have a crease along one of its diagonals. If the mirror is placed along the diagonal the student may be asked to observe what happens when the figure on one side of the diagonal is reflected along the crease (diagonal). This may be confirmed by folding along the crease (and by using pins and mirror-strip). The student may be asked to fold the paper along the other diagonal and to observe what happens. The idea of perpendicularity of two lines can also be taken up at this stage by paper folding.

As for purposes of demonstration, in this chapter, the teacher requires (i) black-board (unruled), (ii) black-board ruled into squares, (iii) black-board instruments: straight edge, compasses, protractors, (iv) models and charts, (v) mirror, (vi) tracing paper, (vii) graph-paper, (viii) scissors, (ix) peg-board or pin board, (x) set-square, (xi) parallel ruler, (xii) Link parallelogram, (xiii) Mirrorstrip (doubled) and long needles attached to corks.

The apparatus necessary for the students consists of (i) a set-square, (ii) a ruler, (iii) scissors, (iv) plain papers for folding, (v) graph paper, (vi) dividers and compasses, (vii) parallel ruler.

V. *Answer to Exercises 2.1.*

(1) Isosceles triangle — one line of symmetry, the bisector of base.
Rectangle — two lines of symmetry, the lines joining the middle points of opposite sides.
Square — 4 lines of symmetry. Two lines as in rectangle, two diagonals.
Circle — any diameter.
Equilateral triangle — Three lines of symmetry, right bisectors of sides.
Parallelogram — No line of symmetry.
Rhombus — Two diagonals.
Isosceles trapezium — The line joining the middle points of parallel lines.

Semi circle — The radius which is the right bisector of the diameter.

(6) The figures mean coupled figures.

VI. List of concepts, sub-concepts, vocabulary, symbols

Concepts: Reflection-mapping, symmetry in a line, perpendicularity, congruence, circle.

Sub-concepts: Fixed points, fixed lines, relation between perpendicularity and parallelism, tangents.

Vocabulary

Reflection-mapping, symmetry, right bisector, perpendicularity, fixed points, fixed lines, congruence of segments, circle, diameter, chord, segment of a circle, exterior and interior of a circle, secant, tangent.

Symbols

$\alpha, \beta, \gamma, \lambda, \mu$ (Reflection mapping in the lines a, b, c, l, m, respectively).

$\perp$ (Perpendicular).

$\Leftrightarrow$ (implies and is implied).

$\Rightarrow$ (implies).

$\cap$ (intersection).

CHAPTER III

ORDER RELATION

I. Aims and objectives of the chapter

In continuation of the first chapter in which the ideas of a point, a plane and a line have been introduced to the students and in which the ideas of segments and congruence have been taken up intuitively, this chapter, which is to be taught to the students of the fifth class, deals initially with the idea of notion of order of points in a line which is a good example of *relation* among the set of points on a line. In this chapter, the ideas of *segment* and *ray* have been introduced as subconcepts. The congruence of segments introduced in this chapter has to be explained to the students in a clear way. It is at this stage some efforts have been made to connect up Geometry with Algebra and thus the language of 'sets' has been made use of at every reasonable opportunity. From this stage onwards the notion of mapping which had been taken up earlier and which is used more prominently in subsequent chapters has been briefly taken up in this chapter. In this chapter, some constructions relating to the equality of segments with the use of geometrical instruments have been treated, in more details and these constructions lead to the marking of points associated with the rational coordinates on a directed line. The concept of one-to-one correspondence of points and numbers (rational numbers only) have also been dealt with, since they are vitally necessary in connection with geometry also.

II. Method of introducing the concepts

The object of this chapter is to introduce the ideas of order relations on a line informally and intuitively.

SECTION 3.1: *Directed line*:

The students should be asked to recapitulate their ideas on the lines (considered in the previous chapter) representing the South, East, North, West directions. The lines may be drawn on the board so that they may run from West to East or vertically upwards or vertically downwards. If we want to go from West to East, we say that we are moving from West in the eastern direction and to indicate the direction, we put an arrow on the line as in the figure (1)

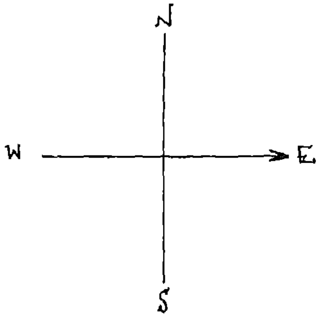


FIG - (1)

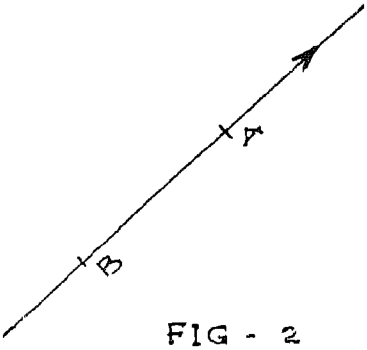


FIG - 2

If, as in figure (2), we go along the line from B to A, we go in the direction of the arrow. If the direction of the arrow is reversed, then we say that the order is reversed. The teacher may cite the example of an weather-cock which determines the direction of the wind. How does a boy come to know the direction of the wind when he flies a kite? The teacher may put the following question to each student and ask them to think at home:

What is the direction of your school-building from your home?

Thus the students who are so far familiar with line extended indefinitely in both directions, will come to know intuitively what a directed line is.

New concept: Directed line.
New vocabulary: Directed line.
New symbol: → (representing a line head).

SECTION 3.2: *Notion of order on a directed line; Betweenness.*

The text provides the students with the idea of position of a point with respect to other points (left or right) on a directed line, where three points A, B, C are chosen. But it would be better if the teacher takes two points A, B first and then explain the relations $A>B$, $A<B$, $A=B$. Since the direction on a line is fixed by an arrow, it should be explained to the students that given two different points A, B on a directed line (with an arrow) then, either A is to the left of B or B is to the left of A, but not both — with reference to a particular figure, in case of the following figure (3).

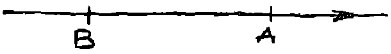


FIG - 3

We write symbolically the relation between the points A, B as $A>B$ or ($B<A$). The teacher may note that the above is true with reference to the above figure. For, if we take just oppositely oriented ray (fig. 4).

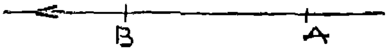


FIG - 4

then the above statements will not be true. But in all cases if $A>B$, then the direction of the line is from B to A. If we now consider three different points A, B, C on the line and A is to the right of B and B is to the right of C, then we describe this situation symbolically as

If $A>B$, $B>C$ then $A>C$

This is called the transitive property which is represented by

$$\{\ A>B,\ B>C \Longrightarrow A>C\ \} .$$

We also say that B lies between A and C.
It is important to illustrate carefully the idea when we write $A=B$ (identical points) . This idea is required later in case of addition of segments. The teacher may note the following also:

Uniqueness of order: For every A and B, one and only one of these three must hold.

A>B, A=B, A<B (law of trichotomy of points).

New concept: Order relations of points on a directed line, betweenness.
New vocabulary: Order relation, betweenness.
New symbols: <, = , > (less than, equal to, greater than)

SECTION 3.3: *Open and closed segments*

Here the students will learn about a segment with a new definition (as given in the text). It may be pointed out to the students that the segment has been defined or rather determined in the earlier chapter by the pair of points. In the light of order relation of points on a line, we define the segment as a set of points as explained below:

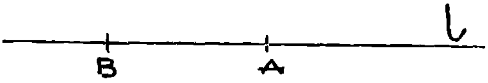


FIG - 6

Let A and B be two different points on a line l. We know that there are many points on the line l in between A and B. The point consisting of A, B and all the points in between A and B constitute a set of points and this set of points is called the segment AB. The points A and B are called the end points of the segment. This new definition enables us to say that the segment AB and the segment BA on the line are the same. The set of points lying between A and B is called the open segment AB, and the set of points lying between A and B together with the points A and B is the closed segment AB. It may be explained that the segment is a subset of a line which we know, in general, extended indefinitely in both directions.

If the direction of the line is marked and if the direction of the line is from A to B, i.e. symbolically, A<B then we say that the segment AB is a directed segment and we denote it by $\overrightarrow{AB}$,

Similarly, the segment BA will be denoted by $\overrightarrow{BA}$ if the direction of the line is reversed i.e. if it runs from B to A.

New concept: Open and closed segments (subconcept)

New Vocabulary: Open and closed segments

New symbols: $A \leqslant P \leqslant B$, $A<P<B$.

SECTION 3.4: *Half lines or rays*

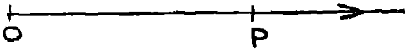


FIG - 7

The students may be asked to look at the figure. The arrowhead on the right of O indicates that the figure consists of all points on the line to the right of the point O and the point itself. The figure demonstrates the idea of a ray which is denoted by $\overrightarrow{OP}$. O is called the starting point or initial point of the ray. When we write $\overrightarrow{OP}$ i.e. when the ray goes from O to P then we mean that the ray starts from O and goes through the point P and runs in the same direction indefinitely. In the figures below (Fig. 8) four different rays are drawn.

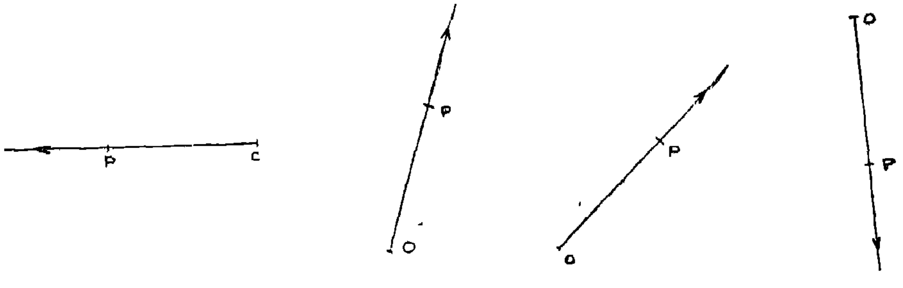


FIG - 8

The student will grasp the ray $\overrightarrow{OP}$ as the set of points P: $P \geqslant O$ on a given directed line. Intuitively, a ray is like a ray of light issuing from a point source of light: it has a starting point (initial point) and extends indefinitely along a line in one direction. To explain an oppositely directed ray let us consider the following figure

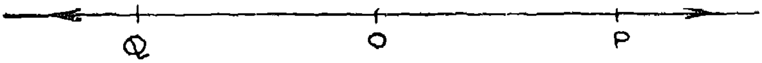


FIG - 9

Here the ray $\overrightarrow{OQ}$ is constituted by the set of points to the left of O as explained in the text.

The teacher may clearly point out the distinction between a ray and a segment. What should be pointed out to the students is that in the notation for a ray, for instance $\overrightarrow{OP}$, the first letter is the initial point and the second letter is one of the infinitely many points through which the ray passes That a fixed point and a direction are always associated with a ray should be clearly explained to the students.

New concept: Half lines, rays

New vocabulary: Half lines or rays

New notation: $\overrightarrow{OP}$ (ray $\overrightarrow{OP}$ with O as fixed point).

SECTION 3.5: The idea of symmetry-line can be understood by the students by means of paper-folding. The teacher will make the students recall all the ideas of reflection mapping associated with the paper folding in the earlier chapter. The teacher will illustrate by paper folding a basic property in connection with rays i.e. given two rays through the point O, there is exactly one line l through O such that the two rays are mapped on to each other by reflection in l.

He will also point out the particular cases when the two rays coincide or when they lie in opposite directions on the same lines.

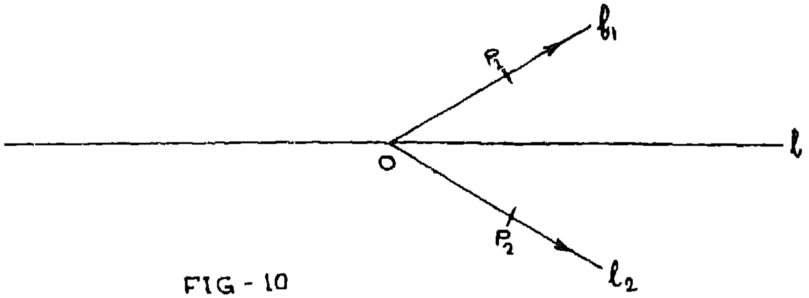


FIG - 10

For this purpose let l_1 and l_2 be two given rays through the common initial point O. We fold the paper in such a way that the ray l_2 falls on l_1 and press the paper to get an impression of the line l between the two rays l_1 and l_2. This line l is called the symmetry line (also called bisector later) between the two rays l_1 and l_2.

In this case any point P_1 on l_1 correspond to some point P_2 on l_2 i.e. $P_2 = \mu (P_1)$. In this way, all the points on l_1 is mapped onto the points of l_2 and thus the line l_2 is the reflection of the line l_1 by the symmetry line l. We write

$$l_2 = \lambda \ (l_1)$$

Children may be asked to obtain the symmetry line of given pairs of rays, by actual paper folding.

New concept: Symmetry line of two given rays.

New vocabulary: Symmetry line.

SECTION 3.6: *Construction of congruent segments.*

In the text, it has been described by means of the figure 85 how a segment OD on a given ray through O can be constructed when OD will be congruent to a given segment AB. The text has provided one way of construction, and here we give below another way of constructing it.

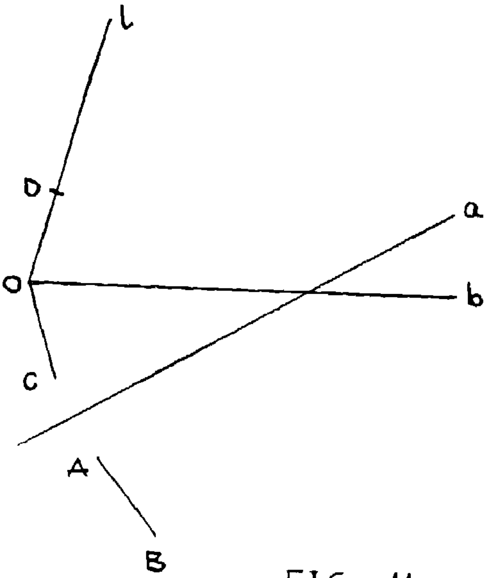


FIG - 11

Fold the paper such that B falls on O. (reflection about the line a) and construct the segment OC which is the map of AB. Finally reflect the segment OC in a suitable line b to get OD on *l*. The practice by the students by means of paper folding is the best way to understand how we can construct a congruent segment on a directed line without using any measuring instrument. By this practice, the students will recapitulate some of the topics like reflections, symmetry line etc.

SECTION 3.7: *Addition of segments.*

In order to determine the sum of two given segments $\overrightarrow{AB}$ and $\overrightarrow{CD}$ which lie on a line and which are in the same direction, we place them side by side on a directed line *l* (*l* is in the same

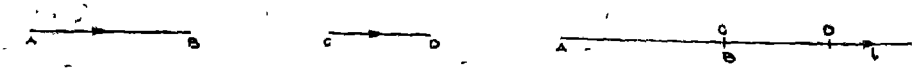


FIG - 12

direction of $\overrightarrow{AB}$ and $\overrightarrow{CD}$) . No matter if we place the segment $\overrightarrow{AB}$ first and then $\overrightarrow{CD}$. Here the points B and C will coincide (idea of coincidence has already been introduced to the students). We say that the sum of $\overrightarrow{AB}$ and $\overrightarrow{CD}$ is the segment $\overrightarrow{AD}$ whose end points are A and D.

In the case of addition of the oppositely directed segments we proceed as follows:

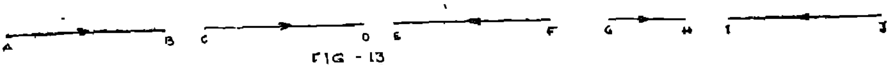


FIG - 13

We see that the segments $\overrightarrow{AB}$, $\overrightarrow{CD}$, $\overrightarrow{GH}$ are directed oppositely to the segments $\overrightarrow{FE}$, $\overrightarrow{JI}$. In this case we add the segments $\overrightarrow{AB}$, $\overrightarrow{CD}$, $\overrightarrow{GH}$ as described earlier, by placing them side by side and starting from a fixed point on the line and get the directed segment $\overrightarrow{OH}$. Now starting from H as initial point we

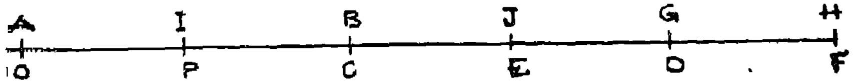


FIG - 14

place the segments $\overrightarrow{FE}$, $\overrightarrow{JI}$ side by side towards the opposite direction, and finally reach at the point P on the line 1. Thus $\overrightarrow{OP}$ is the final directed segment. We shall write

$$\overrightarrow{AB} + \overrightarrow{CD} + \overrightarrow{GH} = \overrightarrow{AH}$$

$$\overrightarrow{FE} + \overrightarrow{JI} = \overrightarrow{FI} \text{ (or } \overrightarrow{HI}\text{)}$$

and $\overrightarrow{AH} + \overrightarrow{HI} = \overrightarrow{AI} = \overrightarrow{AP}$. Inthis connection, the teacher may ask the students to work out the problem no. 1 of Exercise 3.2.

Subconcept: Addition of segments.

SECTION 3.8: *Multiple of a segment.*

In this section the students will come to learn the idea of a segment which is an integral multiple of a given congruent segment. This idea will help them to grasp the idea of presentation of natural numbers by points on a line. Referred to Fig. 14 (ii) we have that the segments OD_1, D_1D_2, D_2D_3,.... are all congruent to each other. In this case we can present any of the segments $D_r\ D_s$ (where r, s can assume any arbitrary value) as an integral multiple of OD_1 as follows:

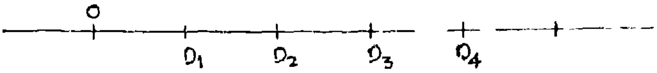


FIG - 14 (ii)

$$OD_2 = OD_1 + D_1D_2 = OD_1 + OD_1 = 2\,OD_1$$
$$OD_3 = OD_2 + D_2D_3 = 2\,OD_1 + OD_1 = 3\,OD_1$$
$$OD_4 = OD_3 + D_3D_4 = 3\,OD_1 + OD_1 = 4\,OD_1$$
$$OD_r = OD_{r-1} + D_{r-1}\,D_r = (r-1)\,OD_1 + OD_1 = r\,OD_1$$

Similarly,

$$OD_s = s\ OD_1$$

Thus when $s > r$,

$$D_r\,D_s = OD_s - OD_r = sOD_1 - rOD_1 = (s-r)\ OD_1$$

The student will verify the above by means of a divider.

SECTION 3.9: *Representation of natural numbers by points on a line.*

Before introducing this topic the teacher should explain the idea behind one-to-one correspondence. He may cite for an example that to each student in the class, a number is assigned. Here the numbers are their roll numbers. The one-to-one correspondence of a number and the houses of a particular street can also be mentioned. Otherwise, the students may confuse numbers to be points on a line. Moreover, it must be explained to them that we cannot associate numbers corresponding to a point at random (as given below).

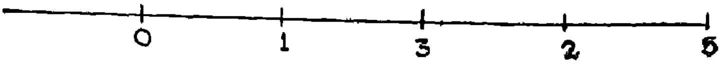


FIG - 15

It should be explained to the students that the positive side and negative side of a number line depends on the choice of the origin (O) on a line. In this connection the teacher may discuss the problem no. 6, sec. 2 in exercise 3.3, no. 2, of Exercise 3.2.
New concept: Representation of natural number as corresponding points on a line, one-to-one correspondence, unit segment.
New vocabulary: One-to-one correspondence, unit segment.

SECTION 3.10: *Division of a given segment into any number of equal parts.*

The practice by the students by means of paper folding is the best way to understand how we can construct a congruent segment on a directed line without using any measuring instrument. By this practice, the students will recepitulate some of the topics like reflections, symmetry line etc. In this connection, the teacher may ask the students to work out problem no. 3, sec. 1 in Ex. 3.3.

SECTIONS 3.11, 3.12 and 3.13: *Representation of rational numbers (fractions) on a line.*

Besides the points 0, 1, 2, 3, ...; —1, —2, —3,.. ... .. we see there are lots of points on the line that have no number assigned to them. Some of them can be plotted if we want to put in the fractions $1/2$, $1/3$, $2/3$, $-1/2$, $-1/3$, $-2/3$ etc. This means the new numbers that we want to put in include all numbers that can be expressed as the ratio $\frac{p}{q}$ of any two integers ($q \neq 0$). We shall call these numbers as rational numbers. Choosing a particular unit segment whose end points correspond to 0 and 1; we know how to divide this unit segment into q equal parts, and taking p such parts will then correspond the points $\frac{p}{q}$. Similarly $-\frac{p}{q}$ can be plotted to the left of the point 0. (If $p > q$, then also we can plot the point $\frac{p}{q}$ or $-\frac{p}{q}$ by means of a divider).

The students so far have learnt intuitively that corresponding to a point there is a whole number and conversely. Now the student will learn about representation of positive and negative rationals on a directed line.

Starting with the line on which points are marked with whole numbers, we can mark other points by dividing the segments into halves, thirds, fourths etc.

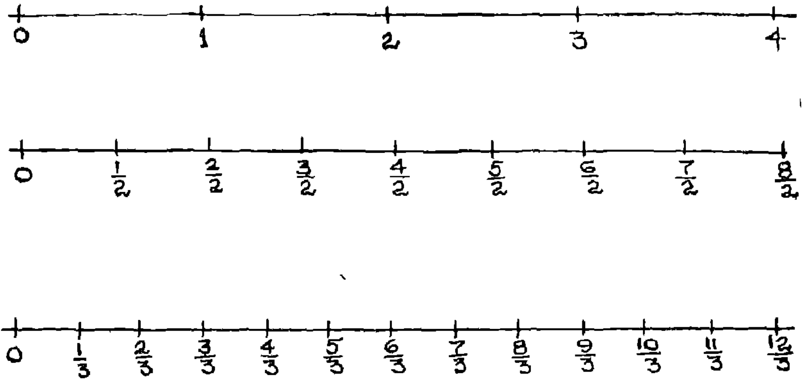


FIG - 16

The above can be represented in a single line as follows:

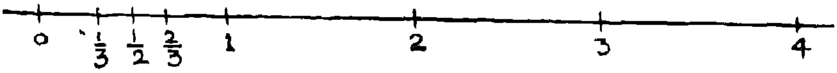


FIG - 17

The teacher may ask the students to put in other numbers on the line.

By this method the students will gather intuitively that there is an infinity of points on any segment of a line. Next, one will choose a given segment as unit segment and say that the length of the segment is equal to one unit. If we place side by side on a line, n segments which are congruent to the unit segment, we say that the length of the segment thus obtained is 'n' units. It may be shown that to get the same length 'n' whether we lay them all in one direction or all of them in the other direction. Naturally, a peculiar situation will crop up when we consider any arbitrary segment on a given line and we are to determine its length.

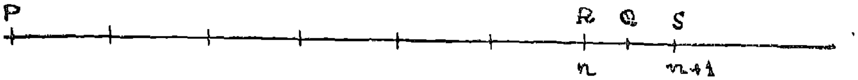


FIG - 18

In this case we lay off segments equal to the unit segment (Fig. 18) starting from the (lefthand) end point and a remainder will be left which is less than one unit segment, i.e. there will be n-abutting unit segments from P to R but the next point S corresponding to (n+1) will fall to the right of Q i.e., (S>Q) in which case we subdivide the unit segment into a suitable number of equal parts, which is generally 10 and mark the smaller unit and obtain the length of RQ approximately correct to the tenth of the unit. So far as actual measurements in the treatment of geometry are concerned, we stop at this stage i.e., correct to one decimal place. The teacher may mention at this stage that the process can be continued to any desired degree of approximation by finer geometrical instruments. He may further add that there may be points of a line which do not correspond to rational numbers. For example the length of the diagonal of a square of unit length is not a rational number but which runs like 1.4142135 .... . The distance between two points P, Q can be obtained by the measuring process described above. We denote the distance between two points P, Q by |PQ| which is obviously a positive number. The teacher will clearly explain the big difference between the segment PQ and the distance PQ. The segment is a geometrical figure, that is, a set of points. The distance is a number, which tells how far P is from Q. The distance does not depend on the order in which the points are mentioned i.e., PQ is the same as QP. It must be emphasised that the length or distance between two points depends on the particular unit chosen. It should be made clear to the students that if we take points P, Q, R in order on a directed line, then the distance

$$|PR| = |PQ| + |QR|$$

The above notions on correspondence lead us to formulate now the idea of what is called a *co-ordinate line.*

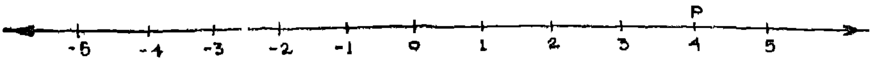


FIG - 19

A directed line with suitable unit distances marked on it in both directions from a fixed origin O, is called a *co-ordinate line.* The number that corresponds to a point P on it is called the coordinate of the point P. We say that $+4$ is the co-ordinate of P if it is at a distance of 4 units towards the right of the point O, and -4 is the co-ordinate of the point Q if it is at a distance of 4 units towards the left of the point O.

We now introduce the idea of the absolute value of a number. We say that absolute value of a number 4 is 4, absolute value of -4 is 4 i.e., the absolute value of any number x is simply the distance between O and the point whose co-ordinate is x on the line, regardless of whether the point lies to the left or right of O We denote this by $|x|$. Next let us consider the examples as in the figure below,

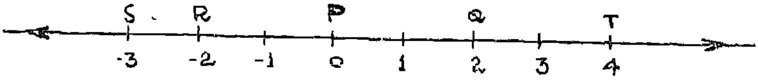


FIG - 20

The points P, Q, R, S and T correspond to the numbers 0, 2, -2, -3, and 4.
PQ = 2 and $2 = 2-0$
QT = 2 and $2 = 4-2$
SQ = 5 and $5 = 2-(-3)$
RT = 6 and $6 = 4-(-2)$

The above examples bring out the fact that the distance between two points, P, Q on it whose coordinates are x, y respectively is the absolute value of $x-y$ and is denoted by $|x-y|$ and in particular, if x and y be both positive and $x>y$ the distance between them is $x-y$. It may be noticed that if we take the pair of points in reverse order, we will get negative numbers which will not certainly represent distance which is always positive number. In order to avoid this difficulty, we have to take the absolute value of the difference of numbers. In this connection, the teacher may work out the problem no. 1, 5 of sec. 1; 3, 4, 5, 7, 8, 9, 10 of sec. 2; 1, 2, 3 of sec. 3 in Ex. 3.3.

New concept: Representation of rational numbers on a line, origin, length or measure of a segment, distance between two points, unit distance, coordinate line, modulus of a number (absolute value).
New vocabulary: Rational numbers, origin, length or measure, distance, coordinate line, modulus (absolute value).
New notation: $|x|$ (modulus of x or absolute value of x).

III. Understanding and skills to be achieved

As already mentioned earlier, this chapter deals with a highly sophisticated form of abstraction. The student has to widen, as in the previous chapter, his grasp over his geometric concepts in an intuitive way. The student has to understand the idea of order relations of points in a line and this will eventually lead him to the concepts of a segment, a half-line or ray. The next important concept which the students should understand is concerned with one-to-one correspondence of points and numbers (upto rational numbers) and in its continution, the idea of coordinates on a directed line.

As for the skills to be achieved, the students have to know (i) the marking of equal segments by using a pair of dividers, (ii) the division of a segment into a number of equal parts. The student should be in a position to ascertain the co-ordinate of a point on a directed line and finally he must be acquainted with the method of measuring distances between two points on a line.

IV. Class-room and student-activities

There is no doubt that the introduction and an adequate treatment of the topics in this chapter is time-consuming, simply because of the abstract nature of the topics. So the duty of the teacher should be to widen the knowledge of geometric concepts in an informal manner rather than deepening students' understanding of the foundations.

A teacher has to be necessarily equipped with the thorough understanding of the matters in an abstract level but will make use of them intuitively and informally. A teacher in unfolding the topics of this unit has to exercise considerable judgement in arranging or assigning the materials so that the students may do the much-needed active learning.

In order to illustrate order relation of points and the correspondence between points and positive integers, the teacher may arrange the students in a row according to their roll numbers in order in one direction and choose any student to be the initial point. A ray may be demonstrated by the teachers as the ray of light issuing from a point source A symmetry line between two rays is to be shown by paper folding and the students may be asked to carry out this paper folding repeatedly. The paper folding is to be used also to illustrate the construction of segment on a given ray by reflecting the given segment in two lines in order . The paper folding is certainly an important activity to which the students at this stage should be drawn into so as to get the point across.

After the teacher has shown to the students the marking of equal segments by using a pair of dividers for a few points on the directed line, he will ask his students to repeat the same for some more points (corresponding to rational numbers) on the line. Next important activity on the part of a teacher and student is the division of a segment into any number of equal parts. A similar treatment may be made in regard to the representation of negative integers and positive and negative rational numbers on the directed line by the teacher and the students. It may, therefore, be possible to point out the links which the study of geometry has with the study of algebra. Every endeavour on the part of teacher is needed so that the students may intuitively feel the arrangement of numbers on a line and thereby, the idea of one-to-one correspondence (rational points) may be driven home to the students. It may be pointed out to them that one-to-one correspondence may be illustrated by other situations. For instance, the teacher may explain to them noting the number of times he has taken attendance in the class by looking to it if each assigned seat in the class is filled up. If he can assign seat to every class, the idea of one-to-one correspondence between the assigned seats in the class room and the students is established.

Before a teacher defines a segment on a ray in the language of sets, it would be proper for him to have a rehashing of the idea of sets in the class. The students may have learnt about sets in (algebra) other classes. It may be explained to him, by way of illustrations that the family to which he belongs is a set of people consisting of his parents, his brothers, sisters and himself also. These people (including himself) are called members of the set.

The geometry class is a set of students: its members are the student himself and his classmates. He belongs to the set and he is a member of the set. He will be called an element of the set. He is a member of his family, of his geometry class and so he is an element of the set of people in his family, and also of the set of his geometry class students. Again both his class and his family contain him and thus we say that both the sets contain him.

We can thus look upon the line as a set of points. In fact, as we will see later that geometrical figures that we talk about can also be looked upon as sets of points. The measurement of length of a segment can be done by the students when the teacher will demonstrate the same in the class. The teacher may mention the fact that the length of the diagonal of a square of unit length is not a rational number but a non-periodic infinite decimal which runs like 1.4142135 .. .. and he may add by way of historical references (which is likely to stimulate curiosity) that this led to the rapid development of mathematics, although it is related that when an ancient Greek mathematician Pythagoras, first recognised that the length of the diagonal of a unit square was not a rational number, he thought that there must be something wrong in God's creation and asked his followers to keep it secret and it was kept as a secret by successive generations of his followers for centuries. Such numbers were called deaf or mute and we have the word *surd* as an expression for such a number still use in the mathematics teaching in high schools (*surd*- in Latin means Deaf-Mute).

After having introduced to the students the coordinate of points on a line, the teacher should consider ample examples to explain the absolute value of a number. He will proceed to obtain the distances between two points whose co-ordinates are x and y. He may begin with three possibilities:

(i) both points correspond to positive numbers
(ii) one point corresponds to a positive value and another a negative value
(iii) both points correspond to negative values.

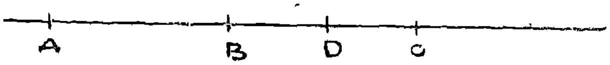
The students will learn that the absolute value of the difference of the numbers gives the distance between these points, irrespective of the order in which we may take the coordinates.

V. Answers to the exercises in the book

Exercise 3.3.

SECTION 1.

1

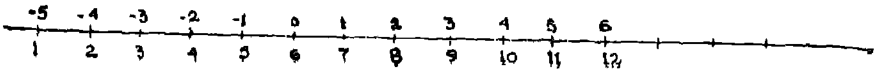


SECTION 2:

3. (d) $x \leqslant 1$ is a ray with initial point 1.
(h) $|x| \geqslant 10$: Two rays oppositely directed with common initial point $x = 10$.
(b) $x = 1$ is a point
(e) $x = -3$ is a point
(f) $|x| \leqslant 3$ is a segment.

6. (a) (i) N/TI (ii) PPT/NT
(b) (i) Rs. 2.64 (ii) Rs. 80.42 (iii) Rs 9.05.

8. (a) Asok



Anil

(b) $|11-3| = 8$ by Anil's scale.

$|5-(-3)| = |5+3| = 8$ by Asoke's scale.

(c) Yes.

Special exercises

I. (a), (b) always correct

(d) if $d>o$, $|d| = d$, $|d| + 1 = d + 1$

2. (a) $O < r$, $|r| = r$

$O > r$, $|r| = -r$

$O = r$, $|r| = o$

VI. **Concepts, sub-concepts, vocabulary and symbols used in the chapter**

Concepts: Order relations of points on a directed line, segment, ray, one-to-one correspondence, points corresponding to any rational number, rational coordinate of a point on a line, symmetry line.

Sub-concepts: Absolute value of a number, distance between two points, surd, length of a segment, addition of segments, coordinate line.

Vocabulary: Order of points on a line, ray, half-line, initial point, unit segment, marking of points on a directed line, one-to-one correspondence, surd, length of a segment, distance between two points, addition of segments, coordinate line, absolute value of a number,

List of symbols used in this chapter together with their meanings

1.	$\rightarrow$	arrow
2.	$>$	greater than
3.	$<$	less than
4.	$=$	equal to
5.	$\geqslant$	greater than or equal to
6.	$\leqslant$	less than or equal to

(These are symbols of order relations)

7. (i) $(B<x<A)$	B is less than x and x is less than A. (The set of all points on the segment excluding the points A and B).
(ii) $(B \leqslant x \leqslant A)$	B is less than or equal to x and x is less than or equal to A. (The set of all points on the segment AB including A and B).
(iii) $(A>B, B>C)$	$\Rightarrow A>C$, (A greater than B, also B greater then C implies A greater than C).
8. $(\overrightarrow{AB})$	Segment $\overrightarrow{AB}$. (This represents directed segment. The direction is from A to B).
9. $(P>O)$	P greater than O. (This symbol represents the set of all points which are greater than the point O).
10. (λ, μ)	Lamda, Mu (These greek-letters have been used to represent reflection-mapping)
11. $\lvert x \rvert$	Modulus of x (The absolute value of x).

APPENDIX

Section I: *A note on the books and journals suggested*:

As the course in geometry set out in the textbooks stands out, in content, from what has been followed hitherto, it is hoped that the books and journals suggested will enable the teachers to follow the line of development in the textbooks and also, will provide a background to their exposition and a direction to their work.

Among the large number of books referred to, one should atleast go through the books by Friis, Jeger, Yaglom, Coxeter, Kutuzov and Moise. The book by Friis may well afford to be a starting one, for it investigates 'some of the commoner properties of transformations applied to plane figures,' based on some knowledge of elementary geometry and then, it proceeds to discuss the 'group' properties of the transformations. Jeger's book, a translation from the original German text, aims at developing transformation geometry from the standpoint of groups. Jeger deals with the notions of rotation, translation and glide reflection as a composition of two or more reflections, without going into the theory of matrices. A similar book, to be cited, in this connection, is that of Yaglom who has dealt with only those transformations that preserve shape and size, or in a word, the *isometries*. The teacher should make it a point to go through the excellent introductory article given in it, with the title 'What is Geometry?' A few chapters of 'Introduction to Geometry' by Coxeter and the book by Blumenthal may provide a background study. A similar argument holds for 'Geometry' by Kutuzov Incidentally, it may be pointed out that Fletcher's handbook 'Some Lessons in Mathematics' gives an integrated view of geometry in one of the chapters of the book. Moreover, some valuable ideas on the teaching of geometry are put forward in this book. Moise's book is a well-known book and should be gone through, even though it is not based on transformations. Although no formal proofs with strict axiomatic rigours are advanced in the textbooks in all cases, as a matter of principle, the teacher should have an acquaintance with all of them. It is, therefore, believed that the teachers might profit immensely by reading

Hilbert's, Forder's, Choquet's and Gugenheimer's books. Freudenthal's book as well as Bell and Fletcher's pamphlet are helpful works in the teaching of symmetry and symmetry groups. The books of Papy and Felix need to be mentioned in this context. The other books on geometry may be read for reference purposes, largely with a view to having a glimpse of what the master-minds like Klein, pioneer in rethinking in geometry, Hilbert, Hadamard, Hessenberg, Weyl, Artin thought and wrote on geometry. It is almost an imperative that a teacher, undertaking a course on geometry considered in the spirit of new mathematics, should have some acquaintance with what new mathematics is It has, therefore, been thought appropriate to include in the above list, the well-known books by Hilbert and Cohn-Vossen, Courant and Robbins, Polya, Felix and Sawyer. We earnestly recommend that the teachers should read as much as they can, the masterly treatises of Hilbert-Cohn Vossen, Courant and Robbins and the recent excellent book by Stein. It is hoped that the books on history, on men and on stories, of mathematics by Bell, Datta and Singh, Newman, Kline will help the teacher considerably to stimulate the curiosity of students, for example, Newman's book (Vol. I) gives information about numbers. Rouse Ball's book is an excellent treatise to be referred to in this context. Skemp's book has been enlisted with the object that the teacher may read a book by a person who is deeply interested in psychological aspects of teaching new mathematics. The display of models by the teacher and their making by the students, as already pointed out in the guides, ought to occupy a major part of the class-room and student activities and the book of Cundy and Rollett is a must in this regard. Rouse Ball's book also contains a discussion on polyhedra. Wenniger's recent publication adds to this category of books Steinhaus's 'Mathematical Snapshots' may prove extremely interesting. As for problems, the other book by Steinhaus and the book by Yaglom *etal* have been suggested. The materials produced by different projects abroad have also been cited, so that the teachers might keep in view of what curricula and materials are being written in other countries. It is with the same aim of acquainting the teachers with an inkling of efforts for curricular reforms in mathematics that the books brought out by organizations O.E.C.D. and UNESCO have been mentioned.

Finally, the journals, generally, contain articles on mathematics, mathematics education, class-room notes, class-room activities, teaching aids and a regular glance through each of them will enable the teachers to keep themselves abreast of the educational researches in their fields of interest.

SECTION II: *Class-room apparatus and materials*:

As already stated, the classroom-activity and student activities form an essential part of the teaching of new mathematics, for the students have to discover facts, by doing and the teacher has to emphasise 'active' learning. All these require that there should be necessary material and apparatus to carry out their own experiments.

The students should be provided with plain papers. Graph paper is essential. Scissors for cutting papers and building models are necesary. Geometrical instruments consisting of the usual set with a ruler and a geo-liner are essential.

For purposes of demonstration, the necessary things are (i) wooden models of polyhedra; thick card or cardboard models for a cube, tetrahedron, pyramid; cardboard models for triangles, quadrilaterals, polygons etc.

(ii) blackboard ruled into squares

(iii) blackboard instruments

(iv) wall papers with patterns and border patterns of various kinds.

In this connection, we make special mention of the book written in 1893 by one enthusiastic countryman of ours—Sundara Row's 'Geometric Exercises in Paper Folding' whose reprint is now made available by Dover publishers. This is an excellent pamphlet which contains, by means of figures, various paper folding exercises that can be done by the students in good paper which can be preserved also. Unfortunately only a few copies were got printed by the author; hence the book had become rare in India. It is gratifying to note that the great mathematician Felix Klein of Germany praised the book for its novel ideas ; this was read by the well-known historian of mathematics D. E. Smith and his colleague Beman of Chicago, who reprinted the book in the first decade of the present century, after obtaining the permission of the author, which was very gladly and readily given.

SOME SUGGESTED BOOKS AND JOURNALS FOR STUDY AND REFERENCE PURPOSES

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